Montana
Office of Public Instruction
Denise Juneau, State Superintendent
opi.mt.gov

Mathematics Model Teaching Unit

Tipi Geometry & Trigonometry

Created by: Marcia Welliever

Grade 10 - Approximate duration: 150 minutes

Stage 1 Desired Results

Established Goals:

Geometric Reasoning Mathematics Content Standard 3: A student, applying reasoning and problem solving, will understand geometric properties, spatial relationships, and transformation of shapes, and will use spatial reasoning and geometric models to analyze mathematical situations within a variety of relevant cultural contexts, including those of Montana American Indians.

- **3.4 Indirect Measurement:** Determine measures of two- and three-dimensional objects and their elements using trigonometric ratios, proportionality, the Pythagorean Theorem, and angle relationships.

IEFA Essential Understanding 1: There is great diversity among the 12 tribal Nations of Montana in their languages, cultures, histories and governments. Each Nation has a distinct and unique cultural heritage that contributes to modern Montana.

IEFA Essential Understanding 3: The ideologies of Native traditional beliefs and spirituality persist into modern day life as tribal cultures, traditions, and languages are still practiced by many American Indian people and are incorporated into how tribes govern and manage their affairs.
Additionally, each tribe has its own oral histories, which are as valid as written histories. These histories pre-date the "discovery" of North America.

Understandings:	Essential Questions:
Students will understand… • how a tipi is constructed. • how the Pythagorean Theorem relates to finding the height of a tipi. • the relationships between radius, height, slant height, base area, lateral area and volume of a cone.	• How does the length of the tipi poles affect the radius, surface area and volume of a tipi? • What are the reasons both past and present Indians use tipis? • How do the properties of a conical solid relate to the advantages and disadvantages of constructing, living in, and transporting a tipi?
Students will be able to… • find the volume, lateral area and surface area of a tipi. • construct a net for the lateral area of a cone. • relate the parts of a cone to a real-world object.	*Students will know…* • the properties and measurable attributes of a cone. • the Pythagorean Theorem. • the formulas for volume, lateral area and surface are of a cone.

Indian Education
Montana Office of Public Instruction
Denise Juneau, Superintendent

Mathematics Grade 10 – Tipi Geometry and Trigonometry (continued) • use a trigonometric ratio to predict the angle of the tipi poles for a given pole length and radius or diameter.	• the definition of the trigonometric ratios. • traditional methods of putting up a tipi. • properties of circle.

Stage 2 Assessment Evidence

Performance Tasks:
Research worksheet on tipis (both past and present). Math worksheets on computing measurements of a specific tipi. Create a model of a tipi using modern materials.

Other Evidence:
Participation in discussion of research findings. Observation of student work. Individual questioning of students.

Stage 3 Learning Plan

Learning Activities:
Day 1:

- Begin the unit by taking 5 minutes to have your class brainstorm the various things they believe to be true about tipis. Write these on the board or on a reference sheet for later. Include any drawings.
- Have the class complete the Tipi Research Worksheet using references from your school's library and/or online resources. There are websites listed in the reference section, if you would like to direct their online work. (If using the older Linda Holley website, I would suggest focusing on the following sections: research area, historic photos & drawings, tipi pattern drawings, linings & ozans, and hide tipis.)

Day 2:

- Begin the day by reviewing the research questions and comparing to the brainstorm list from yesterday. Be sure to cross out any misconceptions or untruths from the list. Students will come back with a wide variety of answers, depending on which tribe or tribes they focused on. This is an important realization for them to have. Discussion of the conical properties as they relate to the tipi shapes (right vs. oblique for some tribes, 3-anchor vs. 4-anchor pole construction) is essential background knowledge for students to be able to complete the mathematical activities.
- (Small groups) Have the students complete the Tipi Math Worksheet to compute the volume of the tipi and surface area of the cover. Be sure to have 3D cone models available to help students to visualize the relationship between the 3D real-life model and the 2D drawings on the worksheet.

Day 3:

- Project: Have the students use modern day materials and a scale of your choosing (ex: 1inch = 1 foot) to construct a model of the tipi described on the Tipi Math Worksheet, or with permission they may want to construct a tipi using an alternate method they researched. Students can work in small groups or individually. Hand out copies of the scoring rubric. Choose a reasonable due date depending on how much you wish students to do in class and how much you expect them to do outside of class.

Extension questions and activities:

- How does changing the angle of the tipi poles with the ground affect the radius, height and volume of the tipi?
- Compute the volume of the tipi that would be created if the tipi poles were the same (l = 26 ft), but the angle they create with the ground is changed to 40 degrees. How would this affect both the volume and the shape of the net?
- For a given surface area and tipi pole length, is there an angle with the ground that will maximize the volume of a tipi?
- Does the traditional 3 anchor method for putting up a tipi lead to the maximum possible volume?
- Find the maximum volume of a cone for a given slant height and radius or height.
- Determine if the Indians used an angle with the ground that maximizes the tipi volume for a given tipi pole length.
- Create pattern for tipi liner on model.

Materials/Resources Needed:
3-dimensional models of a cone
Crow tipi information from Little Big Horn College - http://lib.lbhc.edu/index.php?q=node/80
Blackfeet tipi information from Glenbow Museum -
http://www.glenbow.org/exhibitions/online/blackfoot/main_eng.htm

Please reference the following sites with prudence. Information found on these sites is contemporary and not completely verifiable as to its tribal authenticity.
http://www.tipis.org/
http://www.tipis-tepees-teepees.com

Tipi Research Worksheet
(*Note: tipi, tepee, and teepee are interchangeable)

Using textbooks or internet sources, research the following questions. Your responses are due tomorrow.

1. Which of the Indian Tribes in Montana traditionally used tipis and why?
2. Did any tribes outside of Montana use tipis? If so, what do they all have in common?
3. Most tipi shapes are shown as a right cone with a circular base. Some tribes used slightly adapted shapes. What are they, and what are the advantages or disadvantages to those shapes?
4. Most tipis were covered with tanned buffalo hides pieced and sewn together. Find a diagram or make a drawing to show the net or pattern used to create traditional tipis.
5. Traditional tipis were constructed using poles, typically made of lodge pole pine. Research the number of poles used and methods of erecting the structure. Did all Indian tribes use the same number of poles and the same method? Make a flow chart showing pictures, diagrams and/or verbal descriptions for one method of construction.
6. How did the Indians secure the tipi covers over the poles and to the ground?
7. What methods did the Indians use for temperature control during the various seasons of the year? Which of these methods are still used in modern tipis?
8. How did the Indians control smoke inside the tipi and keep weather outside the tipi?
9. Most modern Indians live in houses, but some also have tipis. For what purposes are modern tipis used? What adaptations with modern materials have been developed and what are their advantages?

Extension questions:

10. Modern homes use drywall, paint, wood or wallpaper as interior decoration. Did traditional tipis have interior decorations? If so, what kind?
11. How were tipis transported? Did the method of transportation have any effect on the size of the tipi used?
12. What furnishings and layouts were used in tipis?
13. Did tipis have a kitchen for cooking?

Tipi Math Worksheet

- One method of constructing tipis is to lash the tops of 3 anchor poles together and place the bottoms of those 3 poles in a triangle where the distance between the poles is the same as the radius of the semicircular cover (some tribes like the Crow use 4 poles). Let the tipi poles overlap each other at the top by 6 feet. More poles (10-20) are laid around a circular base and intersect the anchor poles at the top to create the shell for the tipi. The semicircular cover is then lifted to the top of the tipi, billowed around the poles, and the flaps are overlapped and pinned together to create the tipi. (visit this site for pictures - http://www.tipis.org/images/c_tipi_lg.jpg)

Individually, or in small groups, complete the following:

1. Make a drawing to show the 3-dimensional view of the tipi.
    - Label the intersection of the tipi poles as point V.
    - Label the base of the 3 anchor poles as points P, Q and R.

2. If the tipi poles are about 32 feet long and they overlap on the top by 6 feet, what is the slant height of your cone? _____

   What is the length of the radius of the semicircular cover for the tipi? _____

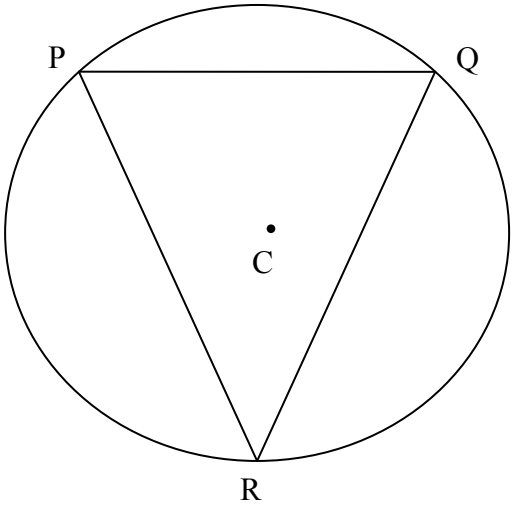
   What is the distance between the bottoms of the anchor poles? _____

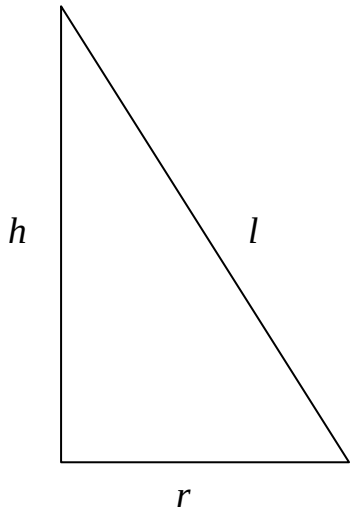
   What kind of a triangle do the anchor poles make on the ground? _____
3. Use the circle below to represent the base of the tipi. (Drawing not to scale) Let the vertices of the triangle represent the bottoms of the anchor poles and C is the center of the circular base.
    - Label the midpoint of side PQ as point M.
    - Draw in triangle CMP and label all of its sides and angles.
    - Justify how you found those sides and angles.
    - What is the radius of the base of the cone? _____
    - Label this radius length on your 3D drawing.
4. Use the right triangle below to represent the one formed by the radius (r), height (h), and slant height (l) of the cone.
    - Use points from the previous drawings to label the vertices of the right triangle below.
    - What mathematical concept can you use to find h in the right triangle below?

      _____
    - Find and label the height of the cone (h) on your 3D drawing.
5. Write the formula for the volume of a right circular cone._____
6. Use your formula to find the approximate volume of the tipi using appropriate units.

   _____
7. Use one of the trigonometric ratios (sine, cosine, or tangent) to determine the angle that the anchor poles make with the ground. Label that angle on the right triangle below.

8. The lateral area of the cone tells us how much material is needed to create the cover of the tipi.
   - Write the formula for the lateral area of a cone._____
   - Use your formula to find the surface area of the tipi using appropriate units._____





Tipi Research Worksheet Answer Key
(*Note: tipi, tepee, and teepee are interchangeable)

Using textbooks or internet sources, research the following questions. Your responses are due tomorrow.

14. Which of the Indian Tribes in Montana traditionally used tipis and why? All of Montana's Indian tribes, including the Sioux, Assiniboine, Crow, Blackfeet, Northern Cheyenne, Salish-Kootenai, Chippewa-Cree tribes used tipis. They were used because they were easy to transport, could be made from local materials, and were adaptable to Montana's varied climate conditions.

15. Did any tribes outside of Montana use tipis? If so, what do they all have in common? Many tribes outside of Montana used tipis. The tribes that used tipis were the Plains Indians who lived and traveled on the Great Plains of North America.

16. Most tipi shapes are shown as a right cone with a circular base. Some tribes used slightly adapted shapes. What are they, and what are the advantages or disadvantages to those shapes? Some Crow and Blackfeet constructed tipis that were in the shape of an oblique cone with a more oval-shaped base. The off-center shape made it possible to arrange the interior of the dwelling differently, make the tipi larger and was a very sturdy construction technique.

17. Most tipis were covered with tanned buffalo hides pieced and sewn together. Find a diagram or make a drawing to show the net or pattern used to create traditional tipis. (See tipi math worksheet.)

18. Traditional tipis were constructed using poles, typically made of lodge pole pine. Research the number of poles used and methods of erecting the structure. Did all Indian tribes use the same number of poles and the same method? Make a flow chart showing pictures, diagrams and/or verbal descriptions for one method of construction. Many tribes used a 3-pole construction technique, where 3 poles were anchored together at the top to begin setup and then other poles were laid over the top to build the support system for the tipi cover. Many Crow and Blackfeet tribes, however, typically used 4-poles to begin construction, and had a more oblique cone tilted away from the door opening.

19. How did the Indians secure the tipi covers over the poles and to the ground? The tipi covers, traditionally made of tanned buffalo hides, were lifted onto the anchor poles and wrapped around the pole "skeleton". Many tipi covers were secured along the slant height with small pegs that went through the covers like button holes. They could be secured to the ground through straps and pegs at the bottom or by placing rocks around the perimeter of the base to secure the tipi against winds and drafts.

Indian Education
Montana Office of Public Instruction
Denise Juneau, Superintendent

20. What methods did the Indians use for temperature control during the various seasons of the year? Which of these methods are still used in modern tipis? Campfires could be built just off-center towards the door opening when heat was needed. Some tipis were constructed with ozans or liners which were like vertical walls added around the perimeter of the tipis' interiors to control drafts. They were typically hung from a rope at about head or shoulder height. During the warm months, the liners could be eliminated and part of the tipi cover could be lifted to create air movement to cool the interior of the tipi. All of these methods are still used in modern tipis, with the addition of modern portable heating devices.

21. How did the Indians control smoke inside the tipi and keep weather outside the tipi? A flap near the top could be narrowed or widened to let smoke escape and control the amount of heat in the dwelling. A rain cap was sometimes added to the construction in wet weather. It allowed the smoke to escape, but "funneled" the rain away from the interior of the tipi. Door covers could be added to provide privacy and access through the oval-shaped opening, while preventing heat exchange.

22. Most modern Indians live in houses, but some also have tipis. For what purposes are modern tipis used? What adaptations with modern materials have been developed and what are their advantages? Modern Indians sometimes use tipis as informal summer dwellings in rural settings. Tipis are used as temporary dwellings during traditional or spiritual ceremonies, such as pow wow gatherings, like the Crow Fair. Many modern tipis are used by both Indian and non-Indian people. Modern updates of materials are canvas tipi covers that can be mildew-resistant, waterproof and weatherproof.

Extension questions:

23. Modern homes use drywall, paint, wood or wallpaper as interior decoration. Did traditional tipis have interior decorations? If so, what kind? Many tipis had painted ozans or liners that displayed Native American artistry.

24. How were tipis transported? Did the method of transportation have any effect on the size of the tipi used? Tipis could be transported by layering the tipi cover and interior belongings on top of the tipi poles and "dragging" them behind dogs or horses. When horses were used, it was possible to use much larger tipis.

25. What furnishings and layouts were used in tipis? Many Indian tribes used beadwork backrests as interior furnishings. These provided support and were a way to display artistic talent as well. Some tipi arrangements had buffalo hide beds which were scattered around the tipi perimeter wall on side away from the door opening.

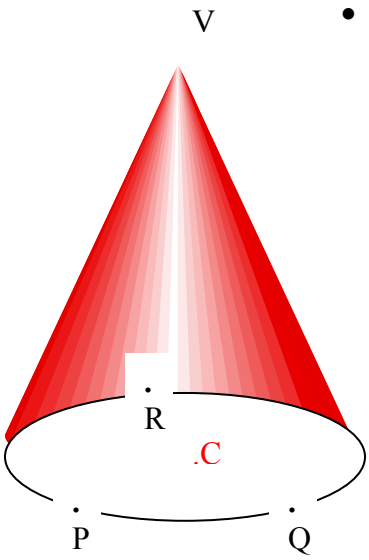
26. Did tipis have a kitchen for cooking? In warmer months, most tribes had a cooking fire outside of the tipis. In cooler months, an interior fire could be used for both heating and cooking.

Tipi Math Worksheet Answer Key

- One method of constructing tipis is to lash the tops of 3 anchor poles together and place the bottoms of those 3 poles in a triangle where the distance between the poles is the same as the radius of the semicircular cover. Let the tipi poles overlap each other at the top by 6 feet. More poles (10-20) are laid around a circular base and intersect the anchor poles at the top to create the shell for the tipi. The semicircular cover is then lifted to the top of the tipi, billowed around the poles, and the flaps are overlapped and pinned together to create the tipi.

Individually, or in small groups, complete the following:

4. Make a drawing to show the 3-dimensional view of the tipi.
   - Label the intersection of the tipi poles as point V.
   - Label the base of the 3 anchor poles as points P, Q and R.



5. If the tipi poles are about 32 feet long and they overlap on the top by 6 feet, what is the slant height of your cone? _26 feet_____

   What is the length of the radius of the semicircular cover for the tipi? _26 feet _____

   What is the distance between the bottoms of the anchor poles? _26 feet _____

   What kind of a triangle do the anchor poles make on the ground? __equilateral___

6. Use the circle below to represent the base of the tipi. (Drawing not to scale) Let the vertices of the triangle represent the bottoms of the anchor poles and C is the center of the circular base.
   - Label the midpoint of side PQ as point M.
   - Draw in triangle CMP and label all of its sides and angles.

$m\angle PMC = 90$
$m\angle MCP = 60$
$m\angle CPM = 30$

$PM = 13\ ft$

$MC = \frac{13}{\sqrt{3}} = \frac{13\sqrt{3}}{3} \approx 7.5\ ft$

$CP = \frac{26}{\sqrt{3}} = \frac{26\sqrt{3}}{3} \approx 15\ ft$

   - Justify how you found those sides and angles.
   - What is the radius of the base of the cone? $CP = \frac{26}{\sqrt{3}} = \frac{26\sqrt{3}}{3} \approx 15\ ft.$
   - Label this radius length on your 3D drawing.

4. Use the right triangle below to represent the one formed by the radius (r), height (h), and slant height (l) of the cone.
   - Use points from the previous drawings to label the vertices of the right triangle below.
   - What mathematical concept can you use to find h in the right triangle below? ___Pythagorean Theorem_____
   - Find and label the height of the cone (h) on your 3D drawing.

9. Write the formula for the volume of a right circular cone.____ $V = \frac{1}{3}\pi r^2 h$ _____

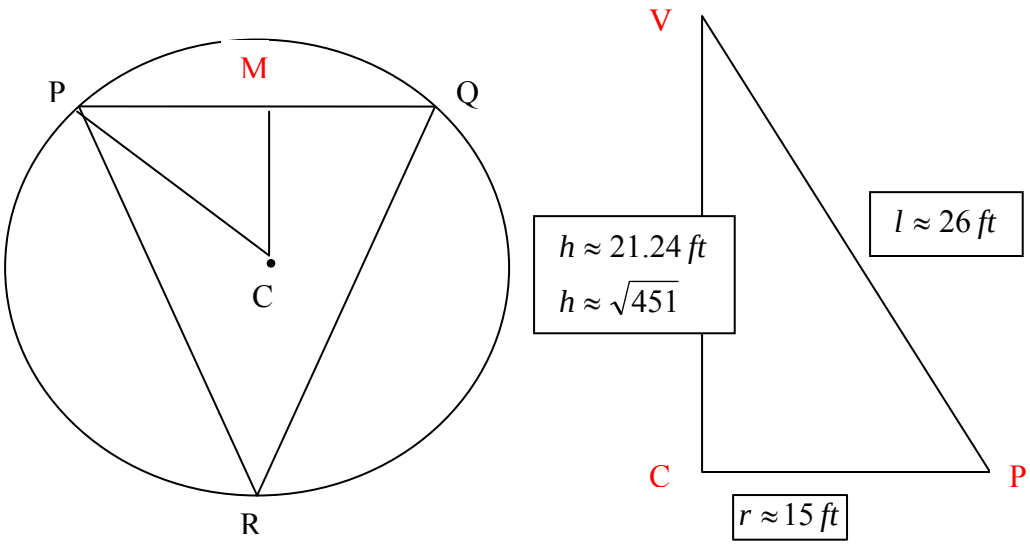
10. Use your formula to find the approximate volume of the tipi using appropriate units.

_____ $V = \frac{1}{3}\pi(15)^2(21.24) \approx 1086\ ft^3$ _____

11. Use one of the trigonometric ratios (sine, cosine, or tangent) to determine the angle that the anchor poles make with the ground. Label that angle on the right triangle below.

$m\angle VPC = \cos^{-1}\left(\frac{15}{26}\right) \approx 54.8^\circ$

12. The lateral area of the cone tells us how much material is needed to create the cover of the tipi.
   - Write the formula for the lateral area of a cone.____ $LA = \pi rl$ _____
   - Use your formula to find the surface area (no base) of the tipi using appropriate units.____ $LA = \pi(15)(26) = 390\pi \approx 1225\ ft^2$ ____



Tipi Model Construction Project

Materials needed: Pattern paper. Canvas for tipi covers. Wooden dowels (small diameter) for tipi poles. Round wooden toothpicks. Liner or ozan material (optional).

- Using a scale of _____, construct a paper template of the net (pattern) needed to construct a model of the tipi described in the tipi math worksheet. Label the pattern with any variables used on the math worksheet. Make a copy of your pattern to turn in to your teacher.
- Using one of your patterns, determine the size of canvas rectangle (to the nearest 1/8 yard) that you will need to construct your tipi cover. Confirm the size needed with your teacher.
- Using the appropriate amount of canvas, cut out your net.
- Determine how many tipi poles you wish to use and how many dowels you will need to construct adequate lengths.
- Use the small wooden toothpicks to lash together the tipi cover.
- Construct a door cover and show smoke flaps. (Liner or ozan are needed for a maximum score)
- Compute the lateral area and volume of the model constructed. Include all formulas used and show all mathematical work. Use appropriate labels. Include this sheet with your pattern page.

	Template for Holistic Scoring of Tipi project
5	Demonstrates complete understanding. All mathematical formulas and work are accurate. 3D model is neat and well-constructed, including door cover, smoke flaps, and liner.
4	Demonstrates considerable understanding. Minor errors in mathematical work. 3D model may have minor flaws or may be missing liner or door cover.
3	Demonstrates partial understanding. Wrong formulas used, but mathematical work is accurate. Pattern (net) used might not yield an accurate model. May have pattern, but no 3D model. Most requirements of task are included.
2	Demonstrates little understanding. Major mathematical errors. Model only partially constructed. Either mathematics is not completed or the model is not attempted.
1	Demonstrates no understanding. Major flaws in any work completed. Major portions of task are not completed.
0	No attempt